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A Review of Multivariate Ordinal Pattern Representations

Workshop of Ordinal Methods: Concepts, Applications, New Developments and Challenges

(ORPATT-22)

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This talk focuses on

- **Ordinal patterns** and its distributions, i.e., **permutation entropy** (PE) introduced by Bandt and Pompe [2]
- Multivariate extensions of ordinal patterns and PE
- Its application in machine learning, in particular classification

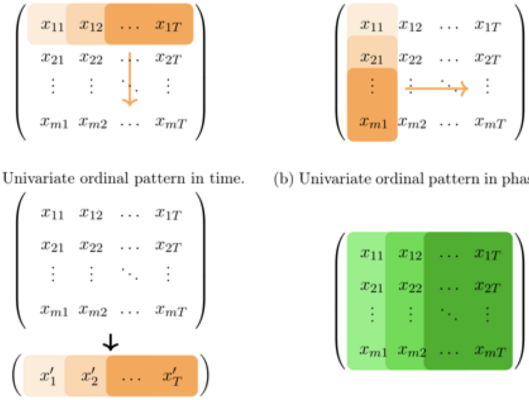
In many fields of applications, multivariate measurements are performed.

Multivariate Ordinal Pattern Representations

$$\begin{pmatrix}1\\2\\0\\1\\0\\1\end{pmatrix} \overset{?}{\geq} \begin{pmatrix}0\\1\\0\\2\\0\\0\end{pmatrix}$$

$$t \qquad\qquad t+1$$

Multivariate Ordinal Pattern Representations



(a) Univariate ordinal pattern in time.

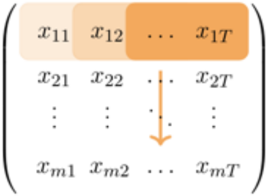
(b) Univariate ordinal pattern in phase space.

(c) Dimensionality reduction.

(d) Multivariate ordinal pattern.

Figure 4.1: Four strategies of MPE determination.

a) Canonical Extensions



(a) Univariate ordinal pattern in time.

Definition (Keller and Lauffer, 2003 [5])

The **pooled permutation entropy (PPE)** of a multivariate time series $X = ((x_t^i)_{i=1}^m)_{t=1}^T$ is defined as **PE of the marginal frequencies** $p_{.j}^{d,\tau} = \sum_{i=1}^m p_{ij}^{d,\tau}$ for $j = 1, \ldots, d!$ describing the distribution of the ordinal pattern and is calculated by

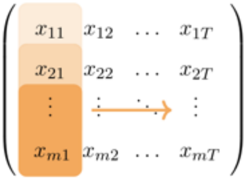
$$\mathrm{PPE}_{d,\tau}(X) = -\sum_{j}^{d!} p_{.j}^{d,\tau} \ln p_{.j}^{d,\tau}. \qquad (1)$$

a) Canonical Extensions

- Multivariate multi-scale permutation entropy (MMSPE) (Morabito et al., 2012 [8]) includes different scales of a time series
- Multivariate weighted permutation entropy (MWPE) (Mohr et al., 2021 [7]) includes different aplitudes in the ordinal patterns

Figure 3.5: Same ordinal pattern of order $d = 3$ (left) for three different motifs (right).

b) Extensions via Spatial Dependencies



(b) Univariate ordinal pattern in phase space.

First, min-max scaling is applied, i.e.,

$$\tilde{x}_t^i = \frac{x_t^i - \min((x_t^i)_{t=1}^T)}{\max((x_t^i)_{t=1}^T) - \min((x_t^i)_{t=1}^T)}. \tag{2}$$

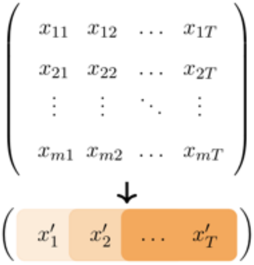
Definition (He et al., 2016 [4])

The Multivariate Permutation Entropy **(MvPE)** of order $d \in \mathbb{N}$ of a multivariate time series $X = ((x_t^i)_{i=1}^m)_{t=1}^T$ is defined as $\mathrm{MvPE}_d(X) = -\sum_{j=1}^{d!} p_j^d \ln p_j^d$, where

$$p_j^d = \frac{\sum_{t \leq T} [(\tilde{x}_t^1, \ldots, \tilde{x}_t^m) \text{ has pattern } j]}{T - (d-1)} \tag{3}$$

with $[x] = 1$ if true, and 0 otherwise, is the frequency of univariate ordinal patterns established over spatial variables.

c) Extensions via Dimensionality Reduction



(c) Dimensionality reduction.

- Reduce the number of spatial variables m to a single dimension by applying an arbitrary dimensionality reduction method. More specifically,

$$
\begin{aligned}
f : \mathbb{R}^{m \times T} &\to \mathbb{R}^{1 \times T} \qquad (4)\\
\begin{pmatrix} x_{11} & x_{12} & \dots & x_{1T} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mT} \end{pmatrix} &\mapsto \begin{pmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1T} \end{pmatrix}.
\end{aligned}
$$

- PE (univariate) can be used directly.

c) Extensions via Dimensionality Reduction

- Multivariate permutation entropy based on Euclidian distance (MPE-EUCL) (Rayan et al., 2019 [9])
- Multivariate permutation entropy based on Manhattan distance (MPE-MANH) (Rayan et al., 2019 [9])
- Multivariate permutation entropy based on normalisation (MPE-NORM) (Rayan et al., 2019 [9])
- Multivariate permutation entropy based on principal component analysis (MPE-PCA) (Mohr et al., 2020 [6])

d) Multivariate Ordinal Pattern Extensions

$$\begin{pmatrix} x_{11} & x_{12} & \dots & x_{1T} \\ x_{21} & x_{22} & \dots & x_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mT} \end{pmatrix}$$

(d) Multivariate ordinal pattern.

Definition (Mohr et al., 2020 [6])

A matrix $(x_1, \dots, x_d) \in \mathbb{R}^{m \times d}$ is associated with **multivariate ordinal pattern (MOP)**

$$\begin{pmatrix} r_{11} & \cdots & r_{1d} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{md} \end{pmatrix} \in \mathbb{N}^{m \times d} \tag{5}$$

of order $d \in \mathbb{N}$ if $x_{r_{i1}} \geq \dots \geq x_{r_{id}}$ for all $i = 1, \dots, m$ and $r_{il-1} > r_{il}$ in the case $x_{r_{il-1}} = x_{r_{il}}$.

d) Multivariate Ordinal Pattern Extensions

$$
\begin{array}{cccccc}
\begin{pmatrix}2&1&0\\2&1&0\end{pmatrix} & \begin{pmatrix}2&1&0\\0&1&2\end{pmatrix} & \begin{pmatrix}2&1&0\\1&2&0\end{pmatrix} & \begin{pmatrix}2&1&0\\0&2&1\end{pmatrix} & \begin{pmatrix}2&1&0\\2&0&1\end{pmatrix} & \begin{pmatrix}2&1&0\\1&0&2\end{pmatrix} \\[1em]
\begin{pmatrix}0&1&2\\2&1&0\end{pmatrix} & \begin{pmatrix}0&1&2\\0&1&2\end{pmatrix} & \begin{pmatrix}0&1&2\\1&2&0\end{pmatrix} & \begin{pmatrix}0&1&2\\0&2&1\end{pmatrix} & \begin{pmatrix}0&1&2\\2&0&1\end{pmatrix} & \begin{pmatrix}0&1&2\\1&0&2\end{pmatrix} \\[1em]
\begin{pmatrix}1&2&0\\2&1&0\end{pmatrix} & \begin{pmatrix}1&2&0\\0&1&2\end{pmatrix} & \begin{pmatrix}1&2&0\\1&2&0\end{pmatrix} & \begin{pmatrix}1&2&0\\0&2&1\end{pmatrix} & \begin{pmatrix}1&2&0\\2&0&1\end{pmatrix} & \begin{pmatrix}1&2&0\\1&0&2\end{pmatrix} \\[1em]
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\[1em]
\begin{pmatrix}1&0&2\\2&1&0\end{pmatrix} & \begin{pmatrix}1&0&2\\0&1&2\end{pmatrix} & \begin{pmatrix}1&0&2\\1&2&0\end{pmatrix} & \begin{pmatrix}1&0&2\\0&2&1\end{pmatrix} & \begin{pmatrix}1&0&2\\2&0&1\end{pmatrix} & \begin{pmatrix}1&0&2\\1&0&2\end{pmatrix}
\end{array}
$$

Figure 5.1: All $(d!)^m = 36$ possible MOPs of order $d = 3$ with $m = 2$ variables.

d) Multivariate Ordinal Pattern Extensions

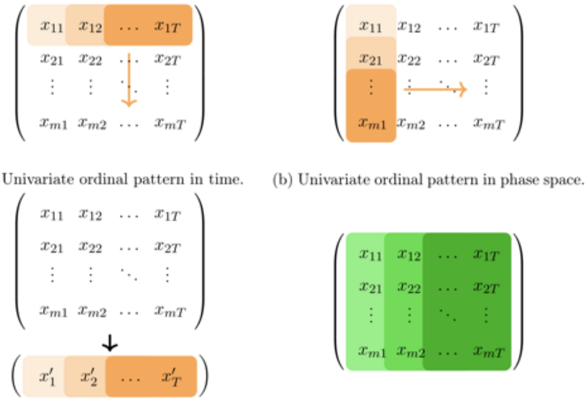
Definition (Mohr et al., 2020 [6])

The **multivariate ordinal pattern permutation entropy (MOPPE)** of order $d \in \mathbb{N}$ and delay $\tau \in \mathbb{N}$ of a multivariate time series $\mathrm{X} = ((x_t^i)_{i=1}^m)_{t=1}^T$ is defined by

$$\mathrm{MOPPE}_{d,\tau}(\mathrm{X}) = -\sum_{j=1}^{d!} p_j^{d,\tau} \ln p_j^{d,\tau}, \tag{6}$$

where $p_j^{d,\tau}$ is the **frequency of MOP** j in the multivariate time series X.

Multivariate Ordinal Pattern Representations



(a) Univariate ordinal pattern in time. (b) Univariate ordinal pattern in phase space.

(c) Dimensionality reduction. (d) Multivariate ordinal pattern.

Figure 4.1: Four strategies of MPE determination.

- PPE
- MMSPE
- MWPE
- MvPE
- MPE-EUCL
- MPE-MANH
- MPE-NORM
- MPE-PCA
- MOPPE
- ...

What to do now?

Representations in Artificial Intelligence

- It's all about **representations**
- As with us, the prediction quality of a data-driven AI or ML model is primarily determined by the data used to train it
- Imagine dividing 210 by 6
- Imagine dividing CCX by VI
- A *good* representation then enables or simplifies a subsequent learning task

When do I need representations for time series?

- Derivation of characteristics in time series analysis (e.g., mean value, variance,...)
- Estimation of the causative factors of dynamical systems
- Extraction of features in ML (classical algorithms can't process time series directly)
- Summarising, *good* representations "help the learner to discover and disentangle some of the underlying (and a priori unknown) factors of variation" [3]


P
Q
R
S
T
ℝ
model

MPE on the UEA MTSC Archive

- Investigation of the relevance of the different ordinal pattern representations for multivariate time series
- Classification task on the UEA Multivariate Time Series Classification (MTSC) Archive
- Higher accuracy means better identification of the the underlying explanatory factors [3]
- The initial benchmarking [1] is with the standard 1-NN classifier with three different distance functions

Results

Data set	PPE	MWPE	MOPPE	1-NN based on EUCL	MANH	NORM	PCA	PCA_2
ArticularyWordRecog.	0.13	**0.14**	0.10	0.13	**0.14**	**0.14**	0.11	0.10
AtrialFibrillation	0.47	0.40	0.40	0.53	0.73	0.60	**0.80**	0.47
BasicMotions	0.55	**0.75**	0.53	0.58	0.38	0.43	0.55	0.58
Cricket	0.24	0.33	**0.36**	0.26	0.31	0.35	0.32	0.29
DuckDuckGeese	**0.36**	0.30	0.32	0.32	0.30	0.30	0.32	0.32
Eigenworms	0.50	0.47	**0.56**	0.47	0.47	0.53	0.49	0.46
Epilepsy	0.50	0.51	0.44	0.41	0.40	0.46	**0.54**	0.51
ERing	**0.49**	0.44	0.41	0.37	0.34	0.30	0.30	0.33
EthanolConcentration	0.29	**0.30**	0.29	0.29	0.29	0.28	**0.30**	0.28
FaceDetection	0.52	**0.53**	0.50	0.52	0.51	0.52	0.51	0.52
FingerMovements	0.54	0.56	0.49	0.54	0.58	0.58	**0.60**	0.57
HandMovementDir.	0.28	0.27	0.32	**0.36**	0.34	0.32	0.27	0.30
Handwriting	0.07	0.05	0.07	0.06	0.06	0.07	**0.08**	0.07
Heartbeat	0.67	0.65	0.65	0.63	0.63	0.62	**0.68**	0.62
JapaneseVowels	0.25	0.23	0.22	0.17	0.25	0.19	**0.26**	0.22
Libras	0.31	**0.37**	0.29	0.27	0.29	**0.37**	0.31	0.28
LSST	**0.20**	0.03	0.19	0.13	0.13	0.13	0.13	0.13
MotorImagery	0.61	**0.64**	0.60	0.55	0.61	0.63	0.57	0.59
NATOPS	0.19	**0.41**	0.22	0.23	0.25	0.24	0.23	0.22
PEMS-SF	**0.75**	0.61	0.56	0.61	0.60	0.64	0.63	0.63
PenDigits	0.22	0.17	0.18	**0.19**	0.18	0.18	**0.19**	0.15
PhonemeSpectra	**0.06**	0.06	0.06	0.06	0.06	0.06	0.06	0.05
RacketSports	0.38	0.33	0.32	0.32	0.32	0.30	**0.38**	0.32
SelfRegulationSCP1	0.59	0.56	0.57	0.61	0.61	0.58	**0.62**	0.59
SelfRegulationSCP2	**0.61**	0.55	0.56	0.55	0.55	0.57	0.53	0.54
SpokenArabicDigits	0.15	0.11	0.14	0.15	0.15	0.15	0.16	**0.17**
StandWalkJump	0.53	0.60	0.47	0.53	0.47	0.53	0.53	**0.67**
UWaveGestureLibrary	**0.24**	0.22	0.21	0.20	0.19	0.19	0.20	0.20

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Results

- As always in ML ... it depends!
- MOPPE is a valuable representation in the case of a small number m of variables and a large length T of the time series
- MWPE should only be used if the amplitude information is of relevance in the application
- MPE-PCA takes into account the correlations of the spatial variables in the dimensional reduction and is thus more suitable than classical distance measures for reduction.
- $\dots$

Conclusion

- In ML, multivariate ordinal pattern representations are interesting because they are intrinsically motivated by interpretable upward and downward movements.
- Nevertheless, the search for the *best possible* representation remains an exciting field of research.

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Thank You

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